MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

EEM2046 – ENGINEERING MATHEMATICS IV

(RE / TE)

9 MARCH 2018
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of five pages including cover page with five questions only.
2. Attempt **ALL** questions. The distribution of the marks for each question is given.
3. Please write down all your answers in the answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1 (30 marks)

a) Evaluate the contour integral

$$\int_C (x^2 + y)dz$$

where C is the straight line segment $y = x$ from $z = 0$ to $z = 1 + i$.

[10 marks]

b) By using Cauchy's Integral Formula or its variant, identify the singular points and evaluate

$$\int_\gamma \frac{e^z}{z^2(z-2)}dz$$

where γ is the positively oriented circle of radius 1 centred at $z = 0$.

[10 marks]

c) Find the Laurent series of

$$f(z) = \frac{1}{z+1} + \frac{1}{z+4}$$

valid for $1 < |z| < 4$ by applying substitution technique. Leave the answer in sigma notation.

[Hint: $\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n$ valid for $|z| < 1$]

[10 marks]

Continued...

Question 2 (15 marks)

a) Given the probability mass function

$$f_X(x) = cx^5, \qquad x = 1,2,3.$$

Find the value of c.

[4 marks]

b) Let X be a random variable with the following probability mass function,

$$f_X(x) = \begin{cases} \frac{1}{21}(x+5), & x = 1,2,3 \\ 0, & \text{elsewhere.} \end{cases}$$

If Y is another random variable related to X by the transformation $Y = X^2$, find $f_Y(y)$.

[5 marks]

c) The joint probability mass function of random variables X and Y is given by

$f_{XY}(x,y)$	$y=1$	$y=2$
$x=1$	1/4	1/8
$x=2$	1/4	3/8

while the marginal probability mass function of X is given by

	$x=1$	$x=2$
$f_X(x)$	3/8	5/8

Find the marginal probability mass function of Y. Are the random variables X and Y independent?

[6 marks]

Continued…

Question 3 (15 marks)

The state transition matrix of a discrete-time Markov Chain with state space {1,2,3,4} is given by

$$\begin{bmatrix} 0 & 0.3 & 0.3 & 0.4 \\ 0.1 & 0.5 & 0 & 0.4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}$$

a) Draw the state transition diagram of the chain.

[4 marks]

b) Decompose the chain into equivalent classes. Determine whether each class is recurrent or transient.

[4 marks]

c) Find $P_{12}^{(4)}$, i.e. the probability of a particle to arrive at state 2 after four steps if the starting state is 1.

[7 marks]

Question 4 (20 marks)

The simplex tableau for a maximizing linear programming (LP) problem is shown below:

Iteration 0

Basic	z	x_1	x_2	x_3	s_1	s_2	Solution
z	1	-3	-2	-1	0	0	0
s_1	0	-1	0	2	1	0	4
s_2	0	1	3	-1	0	1	5

a) Express the LP problem in its standard form, given s_1 and s_2 are the slack variables with $x_1, x_2, x_3, s_1, s_2 \geq 0$.

[4 marks]

b) Find the optimal solution and optimal value by using simplex iteration method. [Hint: Iterations 1 and 2]

[16 marks]

Continued...

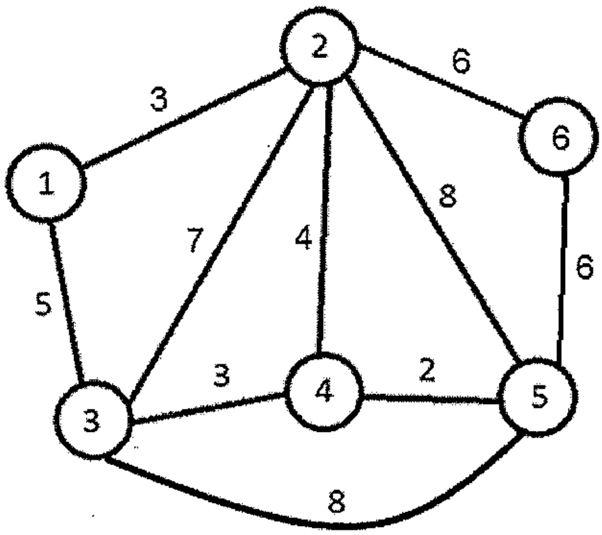
Question 5 (20 marks)

a) Apply Kruskal's algorithm (with detailed steps) to find the minimal spanning tree of the graph below. Hence, draw the minimal spanning tree and specify its weight.

[16 marks]

b) Based on the concept of minimal spanning tree in part a), draw a maximal spanning tree and specify its weight. You **do not need** to show the steps.

[4 marks]



End of Paper